Abstract

Noriyuki Otsubo: The adelic Gaussian hypergeometric function and Anderson-Ihara theory

This is a joint work with Masanori Asakura. The classical Gaussian hypergeometric functions over the complex numbers and their analogues over finite fields are different realizations of the same motives, the former being the complex periods and the latter being the Frobenius traces. Here we only consider those functions $F(a, b; c; x)$ with $c = 1$. By the Euler-Gauss summation formula and its finite field analogue, their special values at $x = 1$ are respectively the beta function and Jacobi sums. We give a definition of the adelic Gaussian hypergeometric function using the adelic étale cohomology of the tower of such motives with respect to levels. This function interpolates all the Gaussian hypergeometric functions (with $c = 1$) over all finite fields. Specializing at $x = 1$, we recover the adelic beta function of Anderson-Ihara; this is an adelic analogue of the Euler-Gauss summation formula.

Masanori Asakura: Frobenius structure on hypergeometric equations, p-adic polygamma values and p-adic L-values

Recently, according to the idea of Dwork, Kedlaya provides certain formula on the Frobenius structure on a hypergeometric equation for $F(a_1, ..., a_n; b_1, ..., b_n; z)$ with $b_i - b_j \notin \mathbb{Z}$. His formula describes the Frobenius matrix on the equation in terms of a product of p-adic gamma values. In this talk, we extend it to the cases $b_1 = ... = b_s = 1, s \leq n$. In such a case, the Frobenius is no longer described by p-adic gamma function, and then we describe it by the p-adic polygamma functions. Since the p-adic polygamma values are linear combinations of p-adic L-values of Dirichlet characters, it turns out that the Frobenius is described by p-adic L-values. Our formula has a geometric application. Let X be a smooth projective family over $\mathbb{G}_m - \{1\}$ such that the Picard-Fuchs equation is a hypergeometric equation. Then it follows from our formula that the Frobenius matrix on the log-crystalline cohomology of the fiber at $z = 0$ is described by p-adic gamma values and p-adic L-values. If the fiber is a total degeneration (hence a mixed Tate motive), then it can be described by p-adic L-values (no p-adic gamma value appears), this is a distinguished application from Kedlaya's result. This is a joint work with Kei Hagihara.

Hironori Shiga: Back stage of $K3$-hypergeometric Arithmetic-Geometric Means

Roman time Platon-school philosopher Plotinus said : the Prima Causa of the world is "the 1". But the European civilization didn't find "0". This concept goes back to the Indian Buddhist philosopher Nagarjuna (c.150 – c.250). These two numbers are the units of two operation systems of C, and they (+ and $\cdot$) are linkaged by the exponential function.

Many interesting number theoretic hypotheses (like the ABC conjecture) appear as the incarnation of the mystery of this linkage. The speaker believes the Gauss AGM theorem is another incarnation. So he is eager to make up a deep and detailed story of AGM-type theorems especially coming from hypergeometric functions (those are extensions of the exponential function).

In this talk he tells about the backstage of new coming AGM theorems induced from a family of $K3$ surfaces on the 4-dimensional Hermitian domain($\cong$ the 4-dimensional type IV domain).

Fuetaro Yobuko: Quasi-F-splitting and klt singularities

Quasi-F-splitting is an extension of the notion of F-splitting, which is defined for schemes of characteristic p. For $K3$ surfaces, F-splitting is equivalent to ordinary and quasi-F-splitting is equivalent to nonsupersingular. These notions are applied to singularities. It is known that two dimensional klt singularities are F-split if the characteristic p is greater than 5, but some of them are not when p is 2, 3 or 5. In this talk, I will present a joint work with Tatsuro Kawakami, Teppei Takamatsu, Hiromu Tanaka, Jakub Witaszek and Shou Yoshikawa showing that two dimensional klt singularities are quasi-F-split in any characteristic.

Shuji Yamamoto: On the Ohno-Zagier type formulas

First I review the Ohno-Zagier formula, which represents the generating function of certain sums of multiple polylogarithms, together with some generalizations. Then I show a proof of it which only uses series manipulations, instead of the original one using differential equation. Finally, if possible, I would like to discuss the reduction modulo p of the truncated series and the relation to the finite multiple zeta values.

Masaki Kato: Functional relations for elliptic q-multiple polylogarithms

The elliptic q-multiple polylogarithm is a common generalization of elliptic and q-multiple polylogarithms. It is also closely related to the double sine function introduced and studied by Shintani and Kurokawa. In this talk, we study functional relations for elliptic q-multiple polylogarithms. We establish p and q-difference relations and inversion formulas satisfied by elliptic q-multiple polylogarithms.

Furusho Hidekazu: p-adic hypergeometric function related with p-adic multiple polylogarithms

I introduce a (maybe new) p-adic analogue of Gauss hypergeometric function, which is constructed from p-adic multiple polylogarithms. I explain my method of a residue-wise analytic prolongation of the function and then show that a p-adic analogue of Gauss hypergeometric theorem holds.

Makoto Kawashima: Hypergeometric series and the Rodrigues' formula

Hypergeometric series is a testing ground for phenomena in irrationality theory. In this talk we shall discuss linear independence of values concerning hypergeometric series. Our results are obtained by a generalization of the classical Rodrigues' formula for the Legendre polynomials. Part of my talk is a joint work with S. David and N. Hirata.

Takeshi Saito: On the Hasse–Arf theorem

The classical Hasse–Arf theorem affirms the integrality of the conductor of an abelian character of the Galois group of a local field. Kato refined this in 1980's by introducing a filtration on the dual group of the abelianized absolute Galois group by using the cup-product with values in the Brauer group and proved a variant of the HA theorem in some cases where the extension of the residue field is inseparable. We reformulate the HA theorem as equivalent conditions for an inequality on the conductor to be an equality. We also discuss a generalization of Kato's theory of Swan conductors.

Ashay Burungale: The BSD conjecture for certain CM elliptic curves and applications

We describe a few results towards the BSD conjecture for CM elliptic curves over the last five years. Some of the results have found applications to classical Diophantine problems such as the cube sum problem. (Joint with M. Flach, S. Kobayashi, K. Ota, C. Skinner, Y. Tian)

Ken Sato: Higher Chow cycles on $K3$ surfaces and their images under the regulator map

The higher Chow group $\mathrm{CH}^p(X, q)$ is a generalization of the classical Chow group $\mathrm{CH}^p(X)$. It satisfies many interesting properties, but its explicit structure is still mysterious for almost all varieties when p is greater than 1.

In this talk, I will explain an explicit construction of non-trivial higher Chow cycles in $\mathrm{CH}^2(X, 1)$ on a $K3$ surface X. Then we show that these cycles generate a subgroup in $\mathrm{CH}^2(X, 1)$ whose rank is at least 18 for very general cases. More strongly, the image of the subgroup in $\mathrm{CH}^2(X, 1)_{\mathrm{ind}}$ is also at least rank 18 for very general cases where $\mathrm{CH}^2(X, 1)_{\mathrm{ind}}$ is the quotient of $\mathrm{CH}^2(X, 1)$ by the images of the intersection product maps.

The key for showing non-triviality of these cycles is calculating the image of the cycles under the Beilinson regulator map. The images of the regulator map are related to a special values of a certain type of normal functions associated with the cycle families. These normal functions have integral representations and satisfy inhomogeneous differential equations which are analogous to integral representations and differential equations of a product of hypergeometric functions.

Yu Katagiri: p-adic properties of division polynomials and algebraic sigma functions

For an elliptic curve over a field, the n-th division polynomial F_n is defined to be a rational function whose zeros are the non-trivial n-torsion points and whose pole is the point at infinity. Silverman considered a sequence $(F_n(P))$ for a fixed rational point P and proved that this sequence has a convergent subsequence in the case that the elliptic curve is defined over a p-adic field. In this talk, we describe the limit of this subsequence explicitly using a special value of a certain sigma function.

Nobuo Tsuzuki: Congruence of Galois representations and application to mirror symmetries of Calabi-Yau in Dwork's families

We introduce a notion of congruence, "mod ℓ reciprocity", between two representations of absolute Galois group of number fields. If one of the images of Galois group is large enough, the two mod ℓ representations will be equivalent. We will give examples of mod ℓ reciprocity in which one of two representations arises from mirror symmetries of Calabi-Yau varieties in arithmetic Dwork's family. We will explain our motivation of the study. This is an ongoing work with Takuya Yamauchi.